

Perturbative stability of SFT-based cosmological models

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Abstract. We review the appearance of multiple scalar fields in linearized SFT based cosmological models with a single non-local scalar field. Some of these local fields are canonical real scalar fields and some are complex fields with unusual coupling. These systems only admit numerical or approximate analysis. We introduce a modified potential for multiple scalar fields that makes the system exactly solvable in the cosmological context of Friedmann equations and at the same time preserves the asymptotic behavior expected from SFT. The main part of the paper consists of the analysis of inhomogeneous cosmological perturbations in this system. We show numerically that perturbations corresponding to the new type of complex fields always vanish. As an example of application of this model we consider an explicit construction of the phantom divide crossing and prove the perturbative stability of this process at the linear order. The issue of ghosts and ways to resolve it are briefly discussed.

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1 Introduction

A new promising class of cosmological models originated from string field theory (SFT) [1] and p -adic string theory [2] has emerged recently and attracts nowadays a lot of attention [3–9]. The main distinguishing feature of such models is the presence of specific non-local operators in the action. There is a strong theoretical reason to consider such non-local interactions. In order to have a good effective model the UV-completeness of the underlying theory is essential and these non-local operators are crucial for making SFT and p -adic string theory UV-complete [1, 2]. Furthermore, such non-local operators have proven to give rise to models with new appealing properties in the context of cosmology,¹ and various interesting results have been obtained recently.

In application to the Dark Energy (DE) problem, one can easily construct models obeying the phantom divide crossing. This is of interest for the present cosmology. Indeed recent results of WMAP [11] together with the data on Ia supernovae and galaxy clusters measurements, give the following bounds for the DE state parameter $w_{\text{DE}} = -1.02^{+0.14}_{-0.16}$. The crossing of the phantom divide as well as the phase with $w < -1$ are not excluded experimentally. Moreover, the experimental data for cosmological epoch corresponding to redshift $z > 1$ leave even more freedom for possible values of w during that phase.

The phantom divide line $w = -1$, corresponding to pure vacuum energy, gives the lower bound of the Null Energy Condition (NEC), whose physical motivation is to prevent instability of the vacuum. Therefore in order to obtain a stable model with $w < -1$ one should construct an effective theory, with the NEC violation, from a fundamental theory which is stable and admits quantization. This is a hint towards SFT inspired cosmological models. It is therefore important to find models where the DE crosses the phantom divide. Among other cosmological models with $w < -1$ and free of instabilities that have been constructed we can mention the Lorentz-violating Dark Energy model [12], the ghost condensation model [13], model with the kinetic gravitational brading [14], model having the dust of the Dark

¹Other non-local models were considered as well in the literature [10] in the cosmological context.

Energy [15], NEC violating models based on the Galileon theories [16], and the brane-world models [17].²

Other results from non-local cosmological models have been obtained in the context of inflation [4, 5]. Such models have in general the remarkable property that slow roll inflation can proceed even in presence of an extremely steep potential [6]. Furthermore they can produce non-gaussian signature for the cosmic microwave background. It has been shown in [4] that the non-linearity parameter which characterizes this non-gaussianity can be large, as opposite to what is described by the canonical single scalar field inflationary models, and observationally distinguishable, for instance, from Dirac-Born-Infeld inflation models.

Recently an attempt to build a non-local generalization of Einstein gravity has also been developed in the literature [7] (see also [21] for an application of the diffusion equation method to non-local gravity models). One result obtained in this direction is the construction of an exact solution which describes a non-singular bounce at the origin and a de Sitter phase in the large time asymptotics.

The main focus of this paper is on a particular class of cosmological models that naturally arise in certain regimes of SFT [3–9]. As it will be explained in details in section 2, such models effectively contain (infinitely) many scalar fields, some of which are characterized by the unusual feature of having complex masses squared. Solutions to this type of models have been analyzed from the cosmological point of view and have proved to be interesting in order to address the questions of the non-gaussianity and the phantom divide crossing. It is however vital to consider the evolution of the (at least linear) perturbations. This step has not been yet covered in full generality for inhomogeneous perturbations in the presence of fields with complex masses squared. It has been explored only to some extent in [6, 8, 22, 23], where space homogeneous perturbations were analyzed. The goal of this paper is to fill this gap.

The paper is organized as follows. In section 2 we review the general properties of this class of SFT derived models. In section 3 we present the specific model that we are going to consider and we show that it admits a class of simple exact solutions in the cosmological background represented by spatially flat FRW universe. The general equations describing the evolution of cosmological perturbations around these exact background solutions are presented in section 4. In section 5 we specialize to the case corresponding to a truncation of the local system to a system with two complex conjugate fields with complex conjugate masses. In section 6 we undertake the numerical study of the perturbations and describe the results obtained. Section 7 describes the issue of ghosts in our model.

2 SFT based non-local models

Non-local models arise naturally in SFT [1] where, under certain approximations, one gets the following space-time action

$$S = \int dx \left(\frac{1}{2} \Phi \mathcal{F}_0(\square) \Phi - V_{int}(\Phi) \right). \quad (2.1)$$

²A remark on stability is necessary. Ghost condensate models look unstable and this is cured by adding higher derivative terms. This in turn implies the presence of other ghosts and related instabilities [18, 19]. Nevertheless, since a cutoff is assumed and the wavelength of this ghost is smaller than the cutoff, the instabilities can be considered irrelevant. The stability claim therefore relies on the fact that a well behaved UV-completion is assumed to exist. This is actually the case in [12], but it seems that ghost condensate models cannot be UV-completed in a usual way [20].

Here $\mathcal{F}_0(z)$ is an analytic function on the complex plane with real coefficients in the Taylor expansion. Φ is a scalar field from the string spectrum and can be for instance the string tachyon. To be more specific consider for example cubic SFT, which may be either bosonic or fermionic. Performing standard SFT calculations one arrives to the action for space-time fields. This consists of two parts: the quadratic part, which is the kinetic term, and the cubic interaction among all the fields. Schematically we can write it as

$$S = \int dx \left(\frac{1}{2} \phi_i K_{ij}(\square) \phi_j - v_{ijk} (e^{-\frac{\beta}{2}\square} \phi_i) (e^{-\frac{\beta}{2}\square} \phi_j) (e^{-\frac{\beta}{2}\square} \phi_k) \right), \quad (2.2)$$

where β is a parameter determined exclusively by the conformal field theory and \square is the d’Alambertian operator. It is natural to expect $\beta < 0$, corresponding to convergent Feynman graphs at large momenta. From this specific example we see the non-local character of the action that is a general feature of SFT based models.³ If furthermore one integrates out all the modes of action (2.2) except one and if only the low mass excitations of the SFT are retained, one obtains

$$S = \int dx \left(\frac{1}{2} \phi (\square - m^2) \phi - \lambda \left(e^{-\frac{\beta}{2}\square} \phi \right)^p \right) \quad (2.3)$$

as an effective action. With a simple field redefinition $\Phi = e^{-\frac{\beta}{2}\square} \phi$ one obviously gets an action of the type outlined above.

The presence of the operator \mathcal{F}_0 is the key modification with respect to a canonical scalar field action and its properties would play the most important role for the resulting model. In SFT this operator can be computed perturbatively by means of the so-called “level truncation scheme” [1]. In general the action of the form (2.1) is just an approximation. In principle a more involved entanglement of non-local form factors and scalar fields would be obtained. An example of the operator \mathcal{F}_0 is given by what one obtains from the tachyon field in the low-level approximation, where one may have $\mathcal{F}_0 = (\square - m^2) e^{2\beta\square}$. A specific expression for \mathcal{F}_0 does not however simplify the following analysis, especially in the case of a curved background, and therefore we keep it general. Moreover, this operator may get modified if we couple our model with other ingredients. Since this may result in possible restrictions on the operator and coupling structures in the theory, it is important to keep \mathcal{F}_0 general.

From the SFT point of view, the full potential $V = -\frac{1}{2}\mathcal{F}_0(0)\Phi^2 + V_{int}$ in (2.1) is expected to have at least two distinct extrema. The scalar field exhibits a non-perturbative transition from one extremum to the other and eventually stops. The best understood processes of this kind in the SFT dynamics is the so called rolling tachyon (see, for instance, [24] and refs. therein), described as a time-dependent kink-like solution. Several solutions in flat background were constructed numerically [24] and in some approximations analytically [25]. Taking the linearization of the potential around a given extremum, say $\Phi = \Phi_0 + \varphi$, a significant analytic progress can be achieved. The linearized action becomes quadratic in the scalar field φ

$$S = \int dx \left(\frac{1}{2} \varphi \mathcal{F}(\square) \varphi \right). \quad (2.4)$$

In the linear approximation, the action will actually have such a form and no other structures are expected.

³Appearance of higher derivatives is not an exclusive feature of this theory. Non-commutative theories, for instance, also have higher derivatives, but these non-local structures are very different.

For the action (2.4) one can establish an equivalence of Lagrangians [9] (notice that this mapping is possible only for the quadratic linearized action and not in presence of a potential, like in (2.1))

$$\varphi \mathcal{F}(\square) \varphi \Leftrightarrow \sum_i \tau_i (\square - J_i) \tau_i,$$

where in the r.h.s. the J_i are roots of the characteristic equation $\mathcal{F}(J) = 0$ and there are as many terms in the series as roots. In other words there is a mapping of a non-local theory with one non-local scalar field to a local theory with many local scalar fields τ_i . To each root is associated one local field, thus in the case of many but finite roots one obtains a genuine local theory with many scalar fields. When infinite roots are present, the non-locality of (2.4) is rephrased in the appearance of an infinite number of local fields. In general the roots J_i may be complex and, in this case, they appear in complex conjugate pairs. As a matter of fact this is not a problem for the theory, since the local fields τ_i are non-physical. They only provide a different mathematical description. It is the original non-local field and the related quantities that must be real, since they represent physical excitations. This translates in requiring that the hermiticity of the local action is preserved when a pair of complex conjugate roots is present. This means that the fields τ_i also enter the action in complex conjugate pairs. The point we want to emphasize is that instead of the usual $\tau\tau^*$ quadratic combination we have terms of the form

$$\tau_i (\square - J_i) \tau_i + \tau_i^* (\square - J_i^*) \tau_i^*.$$

This quadratic expression in general cannot be diagonalized in terms of real fields and this is one of the distinguishing features of such models compared to canonical ones. It is however simpler to work with (infinitely) many local fields rather than one non-local field.

The only thing which is important is the particular vacuum around which the action is linearized and the associated spectrum of the theory. There are many different possible vacua in string theory and the class of models we deal with practically distinguishes these vacua through the form of the operator \mathcal{F} . The latter is in turn characterized by its roots J_i and there may be vacua in which we have no zeros at all (it is believed that the true tachyon vacuum is of this kind with, for example, $\mathcal{F} = e^{2\beta\square}$).

3 Exactly solvable model

Looking for a cosmological scenario and expecting a similar behaviour in a slightly curved background, one is lead to consider a scalar field that interpolates in between of distinct vacua of the potential in a curved background and eventually stops (i.e. φ tends to 0). Then, in the vicinity of the final vacuum, one can construct approximate background solutions for a model where many scalar fields are minimally coupled to gravity and all τ_i tend to 0 [9]. Furthermore, it was explicitly shown in [22] that exists a simple generalization of such a model with (infinitely) many scalar fields minimally coupled to gravity such that the equations of motion in the spatially flat Friedmann-Robertson-Walker (FRW) metric become exactly solvable. This generalization consists of an additional quartic potential

$$\frac{3\pi G}{2} \left(\sum_i \alpha_i \tau_i^2 \right)^2$$

that makes the full equations exactly solvable for specific values of the parameters α_i . (It was argued in [22] that such a modification is obviously the next to the quadratic order expansion of the original potential around an extremum in the cases where the potential is even around this extremum, i.e. in cases where the expansion starts with $V = \frac{1}{2}m^2\varphi^2 + \frac{1}{4!}\lambda\varphi^4 + \dots$). Such an additional term vanishes more rapidly than the other terms of the original model when evaluated on solutions with the scalar fields going to zero. Therefore, in the far asymptotic regime, the results for the model with and without the quartic extra term should coincide.

To be precise, the model of primary concern in the present paper is described by the following action

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} + \frac{1}{g_o^2} \left(- \sum_i \frac{1}{2} (g^{\mu\nu} \partial_\mu \tau_i \partial_\nu \tau_i + J_i \tau_i^2) - \frac{3\pi G}{2} \left(\sum_i \alpha_i \tau_i^2 \right)^2 \right) - \Lambda \right), \quad (3.1)$$

where in addition to the quadratic piece a specific potential of the fourth degree is present. We work in 1 + 3 dimensions with the signature $(-, +, +, +)$, the coordinates are denoted by Greek indexes μ, ν, \dots running from 0 to 3. Spatial indexes are a, b, \dots and they run from 1 to 3. G_N is the Newtonian constant, $8\pi G_N = 1/M_P^2$, where M_P is the Planck mass, g_o is the open string coupling constant, $G \equiv G_N/g_o^2$. $g_{\mu\nu}$ is the metric tensor, R is the scalar curvature, Λ is a constant. The fields are dimensionless, while $[g_o] = \text{length}$.

We impose a specific relation between the parameters, namely $J_i = -\alpha_i(\alpha_i + 3H_0)$, with $H_0 = \sqrt{\frac{8\pi G_N \Lambda}{3}}$. Such a relation, even not being expected to hold always, is the one that one gets in SFT considering the tachyon vacuum in the low-level approximation and with dS background metric [1, 26]. Originally, because of the connection with SFT models, one would set the J_i as the fundamental family of parameters. On the other hand the modification introduced here emphasizes the role of α_i as opposite to J_i . Consider for example one specific J_i . In general there are two possible α associated to it. If one wishes to include both of them in the action one realizes that, due to the specific form of the quartic potential, the two fields corresponding to the two different α are really independent. For instance they cannot be introduced through their linear combination, as opposite to the case of free fields with the same mass. They simply need to be introduced in the action as two general fields, with the special feature that the associated J turns out to be the same. Moreover, as it will become clear, the exact solution is really parametrized by α . Finally, as it will be discussed, for our purposes we are specifically interested in a particular range of values for α . This practically associate in an automatic way one single α_i to each J_i in most of the cases.

In the sequel we are going to explore the case of spatially flat FRW cosmology, with our scalar fields system minimally coupled to the background metric

$$g_{\mu\nu} = \text{diag}(-1, a^2(t), a^2(t), a^2(t)),$$

where $a(t)$ is the scale factor, t is the cosmic time and we shall use the Hubble parameter $H(t) = \dot{a}(t)/a(t)$, denoting with the dot the derivatives w.r.t. the cosmic time. In such a background the modified local action has the following equations of motion

$$\begin{aligned} 3H^2 &= 4\pi G \left(\sum_i (\dot{\tau}_i^2 + J_i \tau_i^2) + 3\pi G \left(\sum_i \alpha_i \tau_i^2 \right)^2 \right) + 8\pi G_N \Lambda, \\ \dot{H} &= -4\pi G \sum_i \dot{\tau}_i^2, \end{aligned} \quad (3.2)$$

and

$$\ddot{\tau}_i + 3H\dot{\tau}_i + J_i\tau_i + 6\pi G\alpha_i\tau_i \sum_j \alpha_j\tau_j^2 = 0, \quad \text{for all } i. \quad (3.3)$$

It is easy to check that they admit the following exact solution

$$\begin{aligned} \tau_i &= \tau_{i0}e^{\alpha_i t}, \\ H &= H_0 - 2\pi G \sum_i \alpha_i \tau_{i0}^2 e^{2\alpha_i t}. \end{aligned} \quad (3.4)$$

This solution is valid for any number of fields (including a single field and infinitely many fields) and for any values of the parameters α_i (i.e. real or complex). We see that if $\text{Re}(\alpha_i) < 0$ for all i then solutions vanish and moreover the quartic term in the potential practically vanishes at sufficiently shorter times and we are left with free fields. Thus for large times the model without a quartic potential is restored and we return to more general SFT based models without requiring any other conditions on the potential to be satisfied. More generally, (3.4) is a solution of linearized Friedmann equations in any non-singular potential with $\tau_i = 0$, $H = H_0$ as a stable fixed point, according to Lyapunov theorem, if $\text{Re}(\alpha_i) < 0$. In our particular setting we deal with a potential for which this solution is exact for the full action. This would simplify the task of considering perturbations, but a generalization of the results to other forms of the interaction seems straightforward.

From the physical point of view, the solutions above describe a very interesting phenomenon, the phantom divide crossing. If there are complex roots J_i , H oscillates around the values H_0 and the equation of state parameter $w = p/\rho$ crosses the value $w = -1$ (p and ρ are the collective pressure and energy density). Indeed

$$w = -1 - \frac{2}{3} \frac{\dot{H}}{H^2}, \quad (3.5)$$

and \dot{H} is an alternating function. Notice that such a crossing is generically not possible with single ordinary scalar field. With two scalar fields one can obtain such a behavior, but one of the two fields must be a phantom (ghost) in this case. In our setup, in presence of complex roots, we also have at least two fields, which are however just an effective description of the original theory with a single non-local field. Moreover, recall that the complex fields enter the quadratic part of the Lagrangian in the following way:

$$\tau_i(\Box - J_i)\tau_i + \tau_i^*(\Box - J_i^*)\tau_i^*.$$

This can be recast in terms of real components as

$$\chi_i\Box\chi_i - \psi_i\Box\psi_i - m_i(\chi_i^2 - \psi_i^2) + 2n_i\chi_i\psi_i, \quad (3.6)$$

where $\tau_i = \frac{1}{\sqrt{2}}(\chi_i + i\psi_i)$ and $J_i = m_i + in_i$. This Lagrangian would be a partial case of the so called B-inflation model [27] provided the “mass” m_i becomes zero for one of the fields, say ψ_i . Also, this resembles the so-called quintom model [28] where two fields (one normal and one phantom) are used as well, but without a quadratic coupling. Because of this coupling we cannot exactly identify the states as normal or tachyons or ghosts, since these notions refer to diagonalized Lagrangians with real fields (in our case we cannot make it diagonal with real fields). This does not mean however that the issue of ghosts is not present in our model. We will return to this question in section 7.

4 Cosmological perturbations

We want to consider the problem of cosmological perturbations for the exact solutions presented in the previous section. We focus on the scalar type perturbations. These are induced by energy density inhomogeneities and are the one that may cause cosmological instability, as opposite to vector type perturbations, that are known to decay rapidly and tensor type perturbations, that correspond to gravitational-wave perturbations of the metric and do not induce any perturbation in the perfect fluid at the linear order. The three kind of perturbations are decoupled and thus one can safely consider them separately [29].

We start presenting the equations for spatially inhomogeneous perturbations of the modified action introduced in the previous section. We refer the reader to [29–31] for the details on the general derivation and to [8] for the conventions adopted here. In our setup the primary quantities are the scalar fields τ_i . It is therefore more transparent to use the fields themselves rather than the notions of energy and pressure for the corresponding components of the stress-energy tensor. It is also important that the spatial dependence of the perturbation variables can be represented as $e^{ik_a x^a}$ for the spatially flat FRW universe, where k_a is a 3-dimensional vector and $k^2 = \delta^{ab} k_a k_b$. With this representation all the relevant information about the space-inhomogeneities are collected into the comoving wavenumber k .

After a number of tedious manipulations the relevant equations for scalar perturbations read [8, 31]

$$\begin{aligned} \ddot{\zeta}_{ij} + \dot{\zeta}_{ij} \left(3H + \frac{\ddot{\tau}_i}{\dot{\tau}_i} + \frac{\ddot{\tau}_j}{\dot{\tau}_j} \right) + \zeta_{ij} \left(-3\dot{H} + \frac{k^2}{a^2} \right) &= \left[\frac{\tau_i}{\dot{\tau}_i} \left(J_i + 6\pi G \alpha_i \sum_l \alpha_l \tau_l^2 \right) \right. \\ &\quad \left. - \frac{\tau_j}{\dot{\tau}_j} \left(J_j + 6\pi G \alpha_j \sum_l \alpha_l \tau_l^2 \right) \right] \left(\sum_k \frac{\dot{\tau}_k^2}{\rho + p} (\dot{\zeta}_{ik} + \dot{\zeta}_{jk}) + \frac{2\varepsilon}{1+w} \right) \\ &\quad + 12\pi G \sum_k \alpha_k \tau_k \dot{\tau}_k \left(\alpha_i \frac{\tau_i}{\dot{\tau}_i} \zeta_{ik} - \alpha_j \frac{\tau_j}{\dot{\tau}_j} \zeta_{jk} \right), \\ \ddot{\varepsilon} + \dot{\varepsilon} H (2 - 6w + 3c_s^2) + \varepsilon \left(\dot{H} (1 - 3w) - 15wH^2 + 9H^2 c_s^2 + \frac{k^2}{a^2} \right) \\ &= \frac{k^2}{a^2} \frac{1}{\rho} \frac{2}{\rho + p} \sum_{k,l} \left(J_k + 6\pi G \alpha_k \sum_j \alpha_j \tau_j^2 \right) \tau_k \dot{\tau}_k \dot{\tau}_l^2 \zeta_{kl}, \end{aligned}$$

where $\zeta_{ij} = \frac{\delta\tau_i}{\dot{\tau}_i} - \frac{\delta\tau_j}{\dot{\tau}_j}$ is the gauge invariant variable for the scalar fields perturbations and ε is the gauge invariant total energy density perturbation. In writing the equations we have introduced the definition $c_s^2 = \dot{p}/\dot{\rho}$ for the speed of sound.

Equipped with these general equations we can consider as a background the exact class of solutions (3.4). Substituting the expression for τ_i , one gets some cancellations and simplifications. Keeping track of all of these, the final result is:

$$\begin{aligned} \ddot{\zeta}_{ij} + \dot{\zeta}_{ij} (3H + \alpha_i + \alpha_j) + \zeta_{ij} \frac{k^2}{a^2} &= \frac{(\alpha_i - \alpha_j)}{\dot{H}} \\ &\quad \left(4\pi G \sum_k \tau_{0k}^2 \alpha_k^2 e^{2\alpha_k t} (\dot{\zeta}_{ik} + \dot{\zeta}_{jk}) + 3H^2 \varepsilon \right), \end{aligned} \tag{4.1}$$

$$\begin{aligned}
& \ddot{\varepsilon} + \dot{\varepsilon} \left(5H + 4\frac{\dot{H}}{H} - \frac{\ddot{H}}{\dot{H}} \right) + \varepsilon \left(6H^2 + 14\dot{H} + 2\frac{\dot{H}^2}{H^2} - 3H\frac{\ddot{H}}{\dot{H}} + \frac{k^2}{a^2} \right) \\
&= \frac{k^2}{a^2} \frac{4}{3} \frac{(4\pi G)^2}{\dot{H}H^2} \sum_{k,l} \alpha_k \alpha_k^2 \alpha_l^2 \tau_{0k}^2 \tau_{0l}^2 e^{2\alpha_k t} e^{2\alpha_l t} \zeta_{kl},
\end{aligned} \tag{4.2}$$

where the equations of state parameter and the speed of sound have been expressed in terms of the Hubble parameter and its derivatives, according to Friedmann equations.

A general analysis of the system given by (4.1) and (4.2) is difficult, even in the case where the background solutions have the simple form (3.4). Many models with multi scalar fields derived from non-local model with a single scalar field and the related problem of cosmological perturbations have already been considered in the literature. Nevertheless, for all the motivations discussed above, the model presented here represent to a certain extent a novelty in this context. In the analysis of the cosmological perturbation we will therefore concentrate on the simplest case that capture the interesting features of the model presented in this work, namely the case of two complex scalar fields.

5 Two complex conjugate roots

When considering complex solutions one should remember that the physical quantities that appear in the original model must be real. In the case of two complex conjugate solutions, labelled by the parameters α_1 and $\alpha_2 = \alpha_1^*$, that we wish to consider, this amount to choose the integration constants in such a way that $\tau_1 = \tau_2^*$. This is exactly the setting we are going to consider. Using the representation $\alpha_1 = \alpha_2^* = \alpha = -\frac{x}{2} + i\frac{y}{2}$ or $\alpha = |\alpha|e^{i\phi_\alpha}$ and using the explicit solutions for $\tau_1 = \tau_2^* = \tau_0 e^{\alpha t}$ one can write all the quantities appearing in the differential equations by means of trigonometric functions. Upon substitution and after some algebra the equations of interest take the form:

$$\ddot{\chi} + \dot{\chi} (3H + x + y \tan(yt)) + \chi \left(3xH + yx \tan(yt) + \frac{k^2}{a^2} \right) = \frac{y\varepsilon H^2}{\cos(yt)}, \tag{5.1}$$

$$\begin{aligned}
& \ddot{\varepsilon} + \dot{\varepsilon} \left(5H + 4\frac{\dot{H}}{H} + x + y \tan(yt) \right) \\
& + \varepsilon \left(6H^2 + 14\dot{H} + 2\frac{\dot{H}^2}{H^2} + 3xH + 3yH \tan(yt) + \frac{k^2}{a^2} \right) = -\frac{k^2}{H^2 a^2} \frac{y\chi}{\cos(yt)}.
\end{aligned} \tag{5.2}$$

Note that all the quantities appearing in the equations are real. All the arbitrary phases have been eliminated. This can be done in all generality corresponding just to a shift in the time variable. The function χ is also real and is related to the original perturbation parameter ζ_{12} for the scalar fields as $\chi = i \frac{8\pi G |\alpha^2| |\tau_0|^2}{3} e^{-xt} \zeta_{12}$. Notice that one can always take $y > 0$ without any loss of generality. In fact one sees from the explicit form of the equations that a change of sign for y can be always reabsorbed as an extra minus factor in the definition of χ . This is not important if we are interested only in separating regions in the parameters space where the perturbations are confined from regions where they are not. Moreover, y just sets the periodicity of the functions appearing in the differential equations and can be therefore conveniently rescaled when studying the numerical evolution of the perturbed system. This corresponds to a change of variable from the cosmic time t

to the dimensionless time coordinate $T = yt$. With such a rescaling of the time variable the perturbative equations are characterized by a set of dimensionless parameters, H_0/y , τ_0 , x/y and k/y , once an extra $1/y$ factor is reabsorbed in the definition of χ . Note that with this redefinition of the time variable one also has $H(t) = yH(T)$. From a practical point of view the rescaling is completely equivalent to set $y = 1$ in (5.1) and (5.2), with a set of parameters, H_0 , τ_0 , x , k , that are now dimensionless. We therefore adopt this choice, setting $y = 1$ hereafter and we still denote the time variable as t . The explicit expression for H in the parameterization adopted is:

$$H = H_0 + h = H_0 - 4\pi G|\alpha||\tau_0|^2 e^{-xt} \cos(t - \phi_\alpha). \quad (5.3)$$

This makes transparent that for certain parameters one can have an expanding universe regime, $H > 0$, with H reaching the constant value H_0 at large times. It is also useful to keep in mind that

$$\dot{H} = -8\pi G|\alpha|^2|\tau_0|^2 e^{-xt} \cos(t)$$

and

$$a = a_o \exp \left(H_0 t - 2\pi G|\tau_0|^2 e^{-xt} \cos(t - 2\phi_\alpha) \right).$$

We stress one more time that hereafter all the quantities appearing in the equations and in the definition of H and a must be understood as rescaled dimensionless quantities. Note also that we emphasized that we are particularly interested in situations where the quartic potential vanishes for large times, corresponding to $x > 0$. As discussed, the rescaling does not arm this point and we can safely work in terms of positive rescaled dimensionless x .

The system of equations (5.1)-(5.2) is in principle ready to be analyzed. An analytic solution for this problem is however not easy to find. In order to proceed with a numerical analysis it is worth noting that there are some special points in the evolution of the system under consideration. Namely, when $t = \pi/2 + n\pi$ one has divergent coefficients appearing in the differential equations. Indeed at these points $\cos(t)$ becomes zero. The numerical integration process may encounter some problem when it reaches these points if the precision is not high enough. Even worse, the system could be really singular. Expanding in series around such points the differential equations read:

$$\begin{aligned} t\ddot{\chi} - \dot{\chi} - x\chi &= -(-1)^n H_n^2 \varepsilon, \\ t\ddot{\varepsilon} - \dot{\varepsilon} - 3H_n \varepsilon &= (-1)^n \frac{k^2}{H_n^2 a_n^2} \chi, \end{aligned} \quad (5.4)$$

where the constants H_n and a_n are respectively the functions $H(t)$ and $a(t)$ evaluated at the n -th singular point. From (5.4) it is quite clear that nothing happens close to the singularity. Numerical integration of this equations and of the non-approximated ones close to the singularity shows that one can pass the singular points without any problem and, if the precision is high enough, all the features of the differential system are captured.

6 Numerical study

We describe here the numerical results obtained starting from the system of equations presented in the previous section and the picture that emerges for the evolution of the perturbations when the different quantities characterizing the background and the perturbations themselves are varied. We particularly focus on distinguishing between situations where the

system remains stable and situations where the perturbations grow making the system unstable. For simplicity we set the constants $G = 1/8\pi$ and $a_0 = 1$. This choice does not affect the evolution of the system in any way.

In order to numerically integrate the system of differential equations (5.1)-(5.2) exact background solutions must be specified. In the case of two complex conjugate roots considered here, this corresponds to specify three real dimensionless parameters, H_0 , τ_0 , x . We specifically focus on the cases of asymptotically vanishing quartic potential, corresponding to positive x parameters. Within this class of solutions, the Hubble parameter H evolves with damped oscillations around the constant value H_0 . In the analysis of the perturbed system we concentrate on the case of cosmologies with expanding universe. This corresponds to ask $H > 0$ in all the stages of the evolution. Such a request, in the region where $\text{Re}(\alpha) < 0$, is easily fulfilled with a balanced choice of α and τ_0 with respect to H_0 . In figure 1 is shown the typical evolution of the Hubble parameter considered in our study of cosmological perturbations.

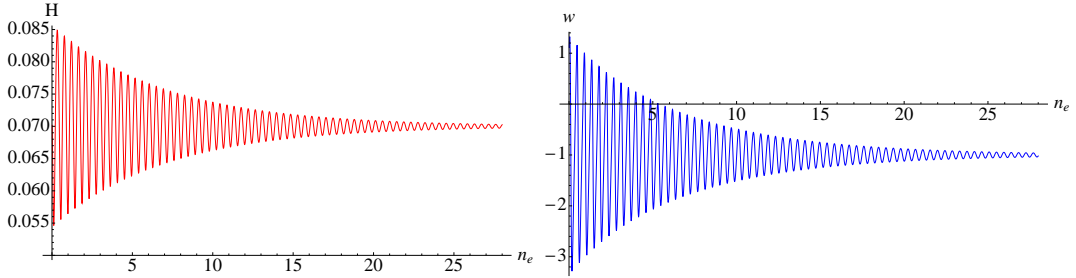


Figure 1. In the left graph, the typical evolution of the Hubble parameter considered in our setting is plotted against the e-folding number. It evolves with damped oscillations towards the asymptotic value H_0 , never crossing the zero. In the right graph, the corresponding evolution for the equation of state parameter w is shown. The crossing of $w = -1$ that characterizes the case of complex roots is evident. The value of the constant parameters considered in the plots are: $H_0 = 0.07$, $x = 0.01$, $\tau_0 = 0.25$.

One comment is necessary in this regard. The constant of integration τ_0 can be in principle tuned at will for the purpose of reproducing the desired behaviour for H . However, from a physical point of view, this choice is not completely arbitrary. Equation (3.2) shows indeed that the value of τ_0 is an important parameter in determining the origin of the Dark Energy content of the model.

One additional parameter that characterizes the perturbed system and that must be specified in order to numerically study the evolution of the perturbations is the comoving wavenumber k . This always enters the differential equations in a specific way. Namely, it always appears in the combination k^2/a^2 and it is therefore mainly relevant in the early stages of the evolution. At large times it become less and less important, due to the suppression given by the increasing scale factor. In particular this combination enters the non-homogeneous term of the differential equations and is therefore an important parameter in setting the coupling between the scalar fields perturbation parameter and the energy density one. Note however that it appears in a non-symmetric way in the differential system (5.1)-(5.2). It only appears in the non-homogeneous term of (5.1), while it enters in a similar way into the left hand sides of the two differential equations.

Finally a set of initial conditions for the differential problem must be specified in order to numerically integrate the differential equations. In particular one would like turn on a

small initial non-zero value for one or both the quantities ε and χ , at some given time. The initial value problem requires also a set of initial conditions for the first time derivative of the two perturbative parameters.

There are mainly two different situations emerging from the study of the differential system (5.1)-(5.2). In both cases one observes that the perturbations decrease to zero at sufficiently large times, giving in principle a well behaved evolution for the system, such as the one shown in figure 2. There are however particular regimes where one must pay attention. While in general one could say that the perturbations evolve with oscillatory

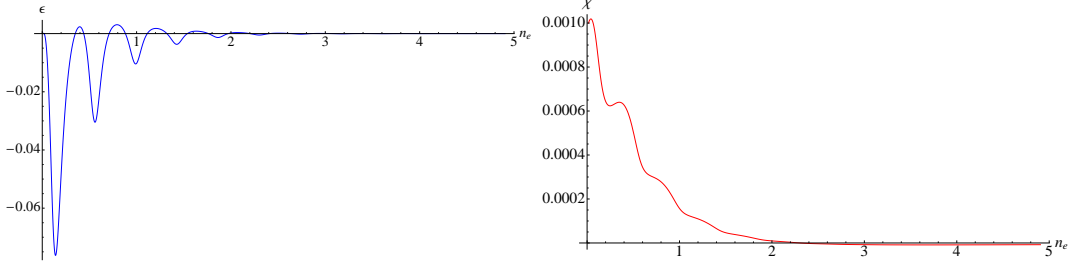


Figure 2. The evolution of ε and χ for $H_0 = 0.07$, $x = 0.01$, $\tau_0 = 0.25$, $k = 0.5$ is shown. The initial conditions are: $\varepsilon(0) = 10^{-4}$, $\dot{\varepsilon}(0) = 0$, $\chi(0) = 10^{-3}$ and $\dot{\chi}(0) = 10^{-4}$.

behaviour according to various possible different paths, but eventually always decreasing to zero value for both ε and χ , in some particular cases one observes a fast initial growth for ε . Precisely, one does not observe divergence for ε . On the contrary, it eventually vanishes, but in the initial stages of the numerical evolution it can exit the perturbative regime.

Indeed when one has non-zero initial conditions for χ , irrespectively of the initial value of ε and its derivative, one naively observes that if $H_0 < 1$ and if k^2/H_0^2 is of the same order of magnitude of $1/\chi$, for values of χ taken in the initial stages of the evolution, ε increases. If the non-homogeneous term in (5.2) is large enough, the energy density perturbation parameter will exceed the range of values where the perturbative regime makes sense. An example of such a situation is depicted in figure 3. This means that if we start with non-zero initial

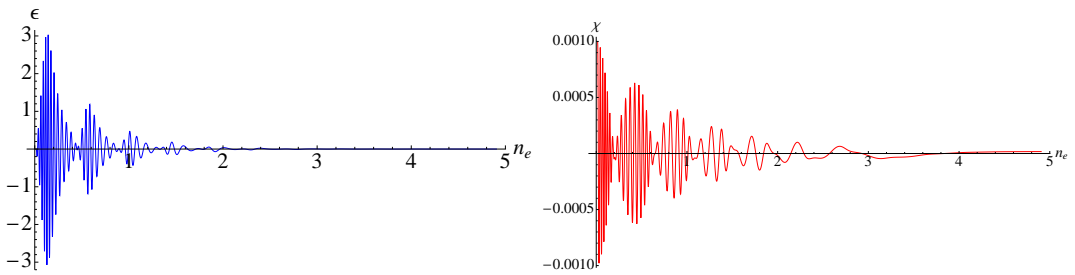


Figure 3. The evolution of ε and χ for $H_0 = 0.07$, $x = 0.01$, $\tau_0 = 0.25$, $k = 15$ is shown. The initial conditions are: $\varepsilon(0) = 10^{-4}$, $\dot{\varepsilon}(0) = 0$, $\chi(0) = 10^{-3}$ and $\dot{\chi}(0) = 10^{-4}$. Even though both χ and ε stay confined and eventually decrease to zero value, ε goes through an initial phase where it shows a fast growth and exits the range of validity of the perturbative regime.

conditions, one can always find a value of k large enough to trigger a fast and large growth for ε . In fact the conditions naively identified for such a phenomenon to occur do just mean

that the non-homogeneous term should remain significantly large in the initial stages of the evolution. The scale factor appearing in the non-homogeneous term, on the other hand gives an exponential suppression of the latter with a rapidity roughly set by H_0 . This can be always compensated with an appropriately large value of k , so as for the balance of the other factors appearing in the non-homogeneous term of (5.2). The exponential suppression of the scalar field solutions, due to the positive values of x considered here, also play a role in this subtle interplay of k with the other parameters. In fact the amplitude of the oscillations covered by ε increases roughly in a polynomial way with k/x . Specifically, for small values of k/x it increases like $(k/x)^2$. When the ratio k/x gets larger this power law behaviour is smoothed out and the maximum range of values spanned by ε increase more slowly with k/x .

In the opposite case, where one sets zero initial conditions for χ and $\dot{\chi}$, the spectrum of perturbations goes to zero without any particular surprise. ε evolves with different oscillatory modes, spanning a range of possible values of the same order of magnitude of the initial conditions and eventually vanishes. In these cases χ is also excited, but only up to values that are practically negligible. Such an asymmetric behaviour dictated by different initial conditions shows that exists a strong transfer from χ to ε , but not in the opposite direction.

One can therefore conclude that for any value of the parameters H_0 , τ_0 , x and for any initial conditions, provided that χ and $\dot{\chi}$ have not simultaneously zero initial value, it is always possible to find a k large enough to obtain a rapid and large growth of ε . This can bring ε out of the range of validity of the linear approximation adopted in the study of the cosmological perturbations. When this happens one cannot rely anymore on the perturbative computations. Since, however, the perturbations remain confined and eventually they all go to zero, it is an interesting question whether this phenomenon signals a real instability of the system or if a more careful analysis, such as considering higher order perturbations, could reveal that the system is indeed stable irrespectively of the value of k .

As it was explained at the end of section 3, the case of (two) complex conjugate roots corresponds to a situation where the phantom divide crossing is realized. The analysis of this section shows that for a large range of parameters the system has a stable behavior under inhomogeneous perturbations and the perturbations are in any case confined and vanish. This allows us to claim that the exact solutions presented in this paper give a stable (at least up to linear perturbations) model with the equation of state parameter oscillating around the value $w = -1$.

7 Taming the ghosts

The numerical analysis of the previous section shows that at the linear order in perturbations the system is stable for a wide range of parameters and initial conditions. Even in cases where the perturbations grow, posing a question on the validity of the perturbative approach, it is enough to wait a sufficiently long time to see that perturbations finally go to zero. One could therefore conclude that there are no divergent modes in perturbations. This however does not mean that ghosts, which are effectively present in our setup, would not trigger instabilities at higher orders in perturbations or when coupled to other fields [32–34]. It is therefore important to understand to what extent ghosts do not pose a threat for the system and our results are applicable.

To clarify the picture we return to the SFT motivation of the model we analyzed. We have considered the next-to-the-linear approximation of SFT dynamics around a given vacuum. There are many different possible vacua in string theory and the class of models

we deal with practically distinguishes these vacua through the form of the operator \mathcal{F} . The latter is in turn characterized by its roots J_i . There may be vacua in which we have no zeros at all (it is believed that the true tachyon vacuum is of this kind for example). Such a vacuum is definitely ghost-free. There may also be vacua with only one root and in this case one has only field, canonical or ghost. When, however, more than one root is present ghosts appear in the spectrum. There are therefore two issues which should be clarified: what is the lifetime of such a vacuum before it gets destroyed by the presence of ghost and whether one can UV complete the model.

As outlined above, on the one hand one might expect that considering higher order perturbations would make manifest ghosts instabilities that are not seen at the linear level. On the other hand, looking at (3.6), it is clear that the ghosts degree of freedom in our model is coupled not only through gravity with everything else, but also directly with χ . The instability is due to the infinite phase space associated to the arbitrarily large negative energies of the ghost that result in a divergent decay rate for the vacuum. However, this instability might be not dramatic in an absolute sense. In fact, the phantom theory is meant to be just an effective description of certain regimes here. The validity of this description therefore depends on whether the ghost instability has the time to develop or not over the relevant cosmological time scale.⁴

We follow the analysis of [32] in order to give an estimate of the domain of validity of our specific model as an effective description with cutoff Λ_{cutoff} . In [32] the allowed decay process involving phantom particles have been analyzed. We are particularly interested to the situation where there is just one ghost field, so as we are assuming in our case. Ordinary particles can decay into ghosts plus other ordinary particles, provided that the effective mass of the final ordinary particles is larger than the original one. When just one ghost is present, this can decay only if in the final states there are at least two ghosts and, at least, an ordinary particle. In order to compute the decay rate of our vacuum it is useful to rewrite the action involving the ghost field with the canonical normalization. We therefore perform the rescaling $\tau/g_o \rightarrow \tau$ in the action (3.1) where, in the case under consideration, the index i denoting the different field run over one complex field τ together with its complex conjugate. We then expand the scalar field Lagrangian in terms of the rescaled real field χ and ψ , with $\tau = \frac{1}{\sqrt{2}}(\chi + i\psi)$ as in (3.6), obtaining

$$\mathcal{L} = \frac{1}{2} \left[\chi \square \chi - \psi \square \psi - m_i(\chi^2 - \psi^2) + 2n\chi\psi - 3\pi G_N \left(\frac{x^2}{4}(\chi^4 + \psi^4) + \frac{2y^2 - x^2}{4}\chi^2\psi^2 + xy\chi\psi(\chi^2 - \psi^2) \right) \right], \quad (7.1)$$

with $m = \frac{y^2 - x^2 + 6xH_0}{4}$ and $n = \frac{y(x - 3H_0)}{2}$. To give a crude approximation on the lifetime of our vacuum, let us set to zero the quadratic term $2n\chi\psi$.⁵ Looking at the quartic part of the Lagrangian we see that several interactions involving phantoms are present. Following the

⁴See for instance [35] for an example where the phantom phase is just an intermediate phase in the cosmological evolution. Some unstable modes propagate out of this phase and some other not.

⁵This is actually a delicate point in our context. In fact setting $n = 0$ is not just a simple choice of parameters. This would correspond to $y = 0$ or $x = 3H_0$, implying $Im(J) = 0$. Therefore taking this specific limit do not correspond at all to a situation with two complex roots for the operator \mathcal{F} , but on the contrary corresponds to a situation with two real coincident roots. For simplicity let us however assume that we can neglect the quadratic term above.

rules above only a few decays are however allowed. In particular the kinematically permitted ones are $\psi \rightarrow \psi\psi\psi$, $\psi \rightarrow \psi\psi\chi$ and $\chi \rightarrow \chi\chi\psi$. The decay rate for a particle at rest for this kind of processes can be written generically as

$$\Gamma = \frac{1}{m_i} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^2 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p_i - p_1 - p_2 - p_3). \quad (7.2)$$

The prefactor is the mass (that is the same in all the processes here) of the particle that decay, \mathcal{M} is the effective coupling constant of the quartic interaction corresponding to the process considered, in our case these are just tree level processes and the factor $|\mathcal{M}|^2$ can be taken out of the integrals. The integrals over the momentum are up to a cutoff energy Λ_{cutoff} , as explained above. The coupling constant of all the processes corresponding to quartic interaction in our Lagrangian is roughly just proportional to $G_N |\alpha|^2 \sim |\alpha|^2 / M_P^2$ and all the phantom and the canonical field considered have the same mass squared, proportional to $|J| \sim |\alpha|^2$. On a purely dimensional analysis basis we can thus estimate the decay rate order of magnitude to be

$$\Gamma \sim \left(\frac{|\alpha|^2}{M_P^2} \right)^2 \frac{\Lambda_{cutoff}^2}{m_i} \sim \frac{|\alpha|^3}{M_P^4} \Lambda_{cutoff}^2. \quad (7.3)$$

If we consider the SFT origin of the class of model discussed, the order of magnitude of α is roughly of the same order as the string mass, which is the inverse of the string length $\sqrt{\alpha'}$, and we can consider it to be related to M_P as $\alpha \sim z M_P$. Here we have introduced a new parameter z , which is just the ratio of the string mass to the Planck mass [36]. z may vary in a wide range since experimentally any mass above, say, 10 TeV is not excluded as a string scale. The natural cutoff coming from SFT is the string mass giving $\Lambda_{cutoff} \sim z M_P$. The decay mean time for the processes considered above expressed in the unit of Hubble time, $H_0^{-1} \sim 10^{60} M_P$, is of the order

$$H_0 \tau_{decay} \sim \frac{10^{-60}}{z^5}. \quad (7.4)$$

It is obvious from the latter that the lifetime of the vacuum can be large and even roughly comparable with the age of the Universe. The price for this is the lightweight string. z , which determines this, can be in the range $10^{-15} \div 1$, since we do not expect strings lighter than 10 TeV from experimental bounds and heavier than the Planck mass from the theoretical perspectives.

Concerning the second question about the UV completion, the following idea might provide an answer in the context of SFT, even if the solution is most likely not unique. The point is that in the class of model we are considering here, in contrast to canonical scalar field models, different vacua can easily have different number and nature of degrees of freedom. One may therefore expect that, apart from the vacuum corresponding to the present model, there is another vacuum which has no ghosts and is generally well behaved. To illustrate this point we can consider the following action

$$S = \int d^4 x \left(\frac{1}{2} \phi(\square - m^2) \phi + v(\mathcal{G}(\square) \phi) \right), \quad (7.5)$$

where \mathcal{G} is a non-local operator and v describes some interaction potential. Now suppose that there are two minima with different second derivatives of the function v only. The kinetic operator would then look like

$$\mathcal{F}_k = \square - m^2 + v_k \mathcal{G}(\square), \quad (7.6)$$

where $v_k = v''(\mathcal{G}(0)\phi_k)$ and ϕ_k are distinct vacuum solutions. A specific function of the form $\mathcal{G} = \alpha(\square - m^2)(e^{\beta\square} - 1)$ shows explicitly that in the vacuum with $v_k = 1/\alpha$ there is only one root for the resulting function \mathcal{F}_k while any other value of v_k generates infinitely many roots and therefore ghosts. Indeed, if $v_k = 1/\alpha$ one has

$$\mathcal{F}_k = (\square - m^2)e^{\beta\square}$$

and this kinetic operator corresponds to one scalar degree of freedom. Even though this is not a genuine SFT derived action, it clearly illustrates how the model can be UV completed.

The bottom line is that the parameters of the model can be such that an effect like the phantom divide crossing is observable and is extended over an appreciable lapse of time, but the eventual stabilization of the model is most likely achieved as a transition to a really stable vacuum and the discussion in the second half of this section describes how this can happen. Here it is important to mention that, as it is discussed in [33, 34], the ghost instability time scale can be finite only if the cutoff is Lorentz violating as in our consideration where we have used a preferred frame. This is not what one expects from the viewpoint of SFT. It is however not excluded that the false vacuum of SFT which contains ghosts could be Lorentz-violating. This question definitely requires better understanding.

8 Conclusions

In this paper we have studied a class of models which describe the late time evolution in the dynamics of scalar excitations in SFT. This class of models admits simple analytic solutions when minimally coupled to gravity. The main distinctive feature is the presence of non-locality arising from the presence of an operator $\mathcal{F}(\square)$ in the action. This non-locality can be translated in a (possibly infinite) set of local fields with masses squared J_i . These masses squared may be complex. The corresponding complex fields enter the Lagrangian in an unusual way, giving rise to a new structure. The main question we have addressed is the influence of such complex fields on the evolution of linear perturbations of the system. In the meantime, we have shown explicitly that these complex fields give a solution describing the crossing of the phantom divide. Therefore stability (or instability) under linear perturbations is essential to understand whether our model can (or cannot) be used as a viable model for the explanation of such a phenomenon.

In order to have good background solutions we must require that the parameters α_i of the model satisfy $\text{Re}(\alpha_i) < 0$. Then the solutions for the scalar fields do not blow up and the Hubble parameter tends to a constant. The most important result is the fact that, having such well behaved solutions, the perturbations related to complex α_i eventually vanish for any values of the other parameters (the comoving wavenumber k , $\text{Im}(\alpha_i)$, the initial data for the background solutions and perturbations). However, we notice that for some range of the parameters the perturbations do exceed the perturbative regime, making the question of validity of the linear approximation unclear. This range of parameters certainly deserves a more extensive analysis including the second order perturbations.

From our analysis it turns out that in a general situation where real and complex J_i are involved, the perturbative spectrum is determined just by the real ones for a wide range of the parameters. Indeed, real values of J_i would generate ordinary real scalar fields. Cosmological perturbations with such scalar fields are well understood and are known to have growing or vanishing modes mainly depending on the values of the wavenumber k . This leads us to state that, since perturbations related to complex J_i almost always vanish, the observable

spectrum of the model is determined solely by real J_i . Thus the crossing of the phantom divide, realized in the case of complex J_i , is perturbatively stable and perturbations do not grow even though the phantom phase is reached.

The presence of ghost-like excitations however is alarming. We have computed an estimate for the vacuum decay rate due to ghosts in the context of our simple model. Admitting that the string mass is not necessarily of the order of Planck mass, one finds a quite large range of values for the lifetime. This can make the described effect of the phantom divide crossing observable. Here we want to stress again that, in SFT based models, the appearance of ghosts is most likely an indication of the fact that there exists another vacuum to which the model evolves. It is exactly the existence of a proper vacuum that is suggested as a way to make the model UV-complete. Even though there can be parameters such a that it is suitable for the present day observations, the model can have relevance for early stages of the Universe evolution, when effects like the phantom divide crossing were also possible.

To summarize we conclude that models of type (3.1) provide interesting results from the point of view of cosmology. Apart from the phantom divide crossing, the appearance of non-gaussianity during the inflation is another fact which makes the class of models considered in the present paper promising from the perspective of future cosmological studies. The results of this paper are generally applicable and one can state that perturbations in similar systems would not be triggered by new type of fields and quantum effects can be suppressed for a long time according to the vacuum decay rate estimate. Therefore, this new class of models is believed to provide new successful ways of resolving some of the cosmological questions. Further aspects, like coupling to other cosmic fluids (CDM, radiation, etc), construction of other potentials, consideration of thermodynamical properties⁶ as well as better understanding of the UV-completion mechanism certainly deserve a deeper study.

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⁶See, for instance, [37] about interesting and promising results concerning the thermodynamics of p -adic strings.

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